

Aberration Invariant Optical/Digital Incoherent Systems

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We have developed a fundamental technique for control of important known and unknown lens aberrations. Control of lens aberrations through traditional means is very difficult in high-performance optical systems. Minimizing aberrations caused by deterministic design errors as well as statistical fabrication errors has often led to costly systems and fabrication techniques. By employing a special-purpose optical phase mask and digital signal processing we can form imaging systems that are invariant, or substantially insensitive, to a number of important lens aberrations.

Key words: wavefront coding, aberrations, optical/digital systems, imaging systems, incoherent imaging

1. Introduction

Control of lens aberrations has traditionally been a very difficult problem. The lens designer often works to determine a lens design where a number of aberrations, such as aberrations due to chromatic aberration, spherical aberration, astigmatism, temperature dependent aberration, fabrication tolerance, etc. are minimized over a broad spatial region in the image plane. The large number of constraints on the lens system invariably leads to designed systems with a large number of tightly specified optical elements. In effect, the number of degrees of freedom of the lens design (optical surfaces and spacings) are increased in order to satisfy the constraints (minimized aberrations). If the number of constraints on the optical system can be decreased, or the number of lens aberrations that can be tolerated is increased, then it should be possible to develop simplified and/or less costly lens systems. Wavefront coding can be used to reduce these constraints.

Wavefront coding is a fundamental technique where a special-purpose optical mask is used to make the optical system invariant, or substantially insensitive, to a large number of important lens aberrations. The direct image formed from a wavefront-coded system is not, however, a clear sharp image. Digital processing of the direct or "intermediate" image is used to restore the spatial resolution to that approaching the diffraction limited system.¹⁻³⁾

The next section describes aberration-invariant systems in terms of invariance to misfocus, or extended depth of field systems. Experimental extended depth of field images are given. Following sections describe aberration invariant properties of extended depth of field systems in terms of ambiguity function analysis. Simulated aberration-invariant imaging results are shown that support this analysis.

2. Focus-Invariance

Wavefront coding for focus-invariance is a new technique in which an optical mask and digital processing are matched to produce imagery with a very large depth of field.^{1,3)} The optical mask alters a traditional incoherent

optical system so that the resulting pointspread functions (PSFs) and optical transfer functions (OTFs) are invariant, or substantially insensitive, to the effects of misfocus. In addition, the optical mask forms OTFs that have no spatial frequency regions of zero power. These two qualities make it possible for a *single* digital filter to be used to restore the spatial resolution of the raw or intermediate focus-invariant imagery. This digital filter is fixed and has no *a priori* knowledge of the amount of geometrical misfocus of the specific scene, or of the characteristics of any objects in the scene.

Examples of focus invariant imagery are given in Fig. 1. This figure is a comparison between the traditional method of achieving a large depth of field, and our new method. Although the depth of field of both systems are equivalent, the F/# of the traditional system is approximately a factor of six larger than the wavefront-coded system. The wavefront-coded system is an F/7.3 system, while the stopped-down traditional system is an F/44 system. This small aperture traditional system captures less than 3% of the optical power captured by the wavefront-coded system. Increasing the size of the aperture of the traditional system noticeably blurs the foreground of the image. The effective F/# can be increased or decreased from this given factor of six depending on the particular optical mask used.

Misfocus can, in general, be considered an aberration. Intuitively we know that if misfocus can be controlled, then so too can be chromatic aberration since chromatic aberration is merely a change in focus as a function of color or wavelength. Astigmatism, where the focal length of a lens appears larger in one plane than in the orthogonal plane, also can be controlled by a system invariant to misfocus. Spherical aberration can be considered a misfocus of adjacent lens zones. If each zone of the lens remains in focus than intuitively spherical aberration can also be controlled. Since coma is not directly related to misfocus, as the above aberrations can be considered, coma is not easily controlled with a focus-invariant system. The effect of more complicated aberrations can be analyzed through the ambiguity function.

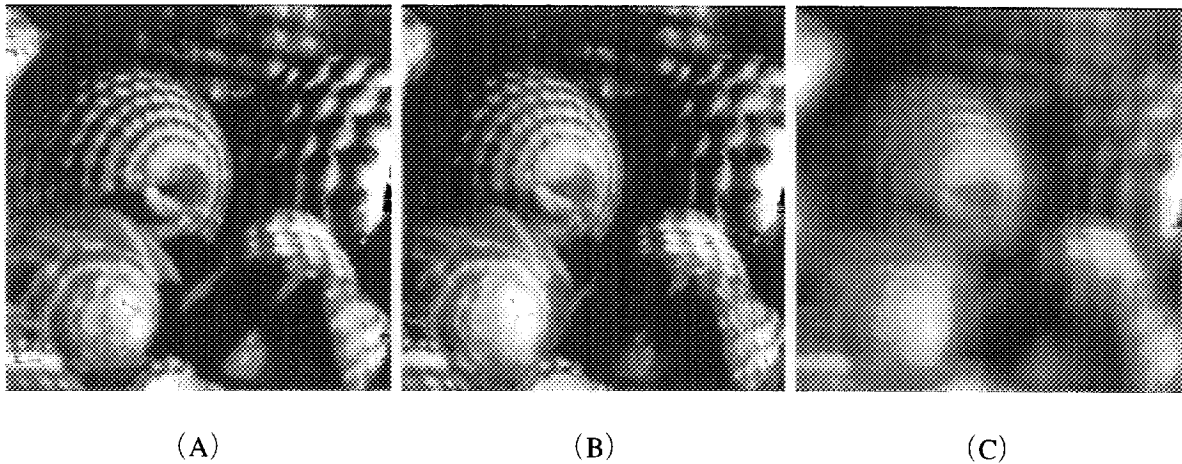


Fig. 1. Comparison of a traditional imaging system with a large depth of field (A), and a focus-invariant imaging system (B). The intermediate or raw image from the focus-invariant system is given in (C). The traditional imaging system (A), is an F/44 system while the focus-invariant system (B) is an F/7.3 system. This reduced aperture of the traditional system allows less than 3% of the optical power captured by the focus-invariant system.

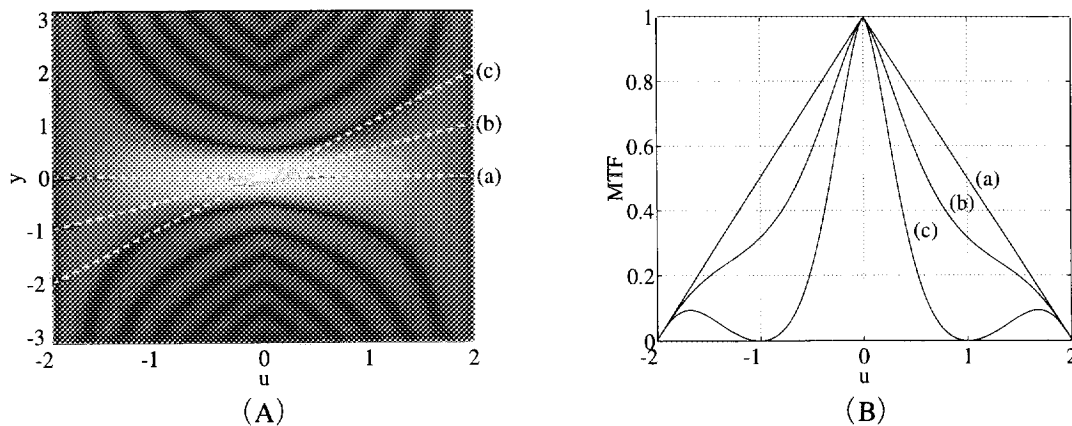


Fig. 2. Ambiguity function related to a rectangular aperture one-dimensional system (A), and the MTF slices (B). The horizontal axis u is in terms of normalized spatial frequency.

3. Aberration-Invariance

An aberration-invariant system is a system that is substantially insensitive to some given amount of aberration. This aberration can be known or unknown, measured or unmeasured. The aberration can be caused by systematic design flaws, or through loose system fabrication tolerances. Such aberration-invariant systems are more general versions of focus-invariant systems.

Our aberration-invariant systems are designed through the use of ambiguity function analysis.¹⁾ The ambiguity function, originally used in the context of radar signal processing, is a powerful analytic representation of the OTF as a function of misfocus.⁴⁾ Properties of the ambiguity function^{5,6)} allow rapid determination of the effect of lens aberrations on the OTF.

The ambiguity function related to a rectangular aperture one-dimensional incoherent optical system is given in Fig. 2(A). The projection of radial lines of this ambiguity function onto the horizontal u -axis determine the OTF of the optical system as a function of misfocus.⁴⁾ Slices of this

ambiguity function are given in Fig. 2(B). Increasing the slope of the radial line is equivalent to increased misfocus. The horizontal axis of the ambiguity function represents the OTF with no misfocus, or the infocus OTF.

An important ambiguity function property is the convolution/multiplication property.^{5,6)} This property is analogous to the convolution/multiplication property found in Fourier analysis. Multiplication in one domain translates into convolution in the other domain. In terms of ambiguity functions, vertical slices or columns of the ambiguity function related to the product of two functions is given by the convolution of the same column of the ambiguity functions related to each of the component functions. Related to Fig. 2(A), a column of the ambiguity function is given by the horizontal axis u equal to a constant and represents the OTF of the optical system at a specific spatial frequency for all values of misfocus.

To determine the effect of aberrations on a specific given system, the ambiguity function related to the observation is used, one column at a time, to convolve with the same representative column of the ambiguity function of the

given system. Or, the OTFs for the two systems evaluated at one spatial frequency as a function of misfocus are convolved, one spatial frequency at a time. This column by column convolution procedure will produce the effective ambiguity function for the aberrated system.

Based on this ambiguity function approach, we can show that optical systems with ambiguity functions that are broad in the vertical or column dimension are less sensitive to aberrations than systems whose ambiguity functions are narrow in the vertical dimensions. A system

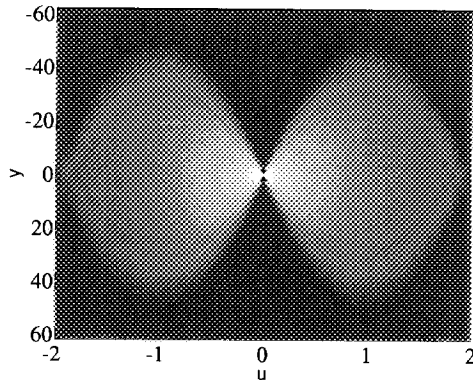


Fig. 3. Ambiguity function related to a one-dimensional aberration-tolerant optical system. Notice that the vertical scale is far larger than that of Fig. 2.

whose ambiguity functions is narrow vertically, and therefore a system with maximum sensitivity to aberrations, is the traditional optical system with the ambiguity function given in Fig. 2. A system with an ambiguity function that is broad vertically, and is therefore less sensitive than traditional optical systems to aberrations, is given in Fig. 3. This ambiguity function is related to a cubic-phase modulation (CPM) focus-invariant system.¹⁾ Comparison with 2(A) also shows that modulation TF (MTF) slices of the CPM system at the same angles as in 2(A) will yield substantially identical MTFs. Note the increased vertical scale on Fig. 3 compared to that of Fig. 2.

The actual degree of insensitivity of a specific system to a given aberration is directly related to the vertical width of the aberration-related ambiguity function. Misfocus-related aberrations produce vertically narrow as well as sheared ambiguity functions. Therefore, these types of aberrations should be easily controlled by focus-invariant systems. Coma related aberrations produce, however, ambiguity functions that are much broader vertically than the misfocus-related aberrations. Therefore, coma related aberrations are more difficult to correct with focus-invariant systems. In order to quantify the aberration tolerant behavior of this system we have simulated off-axis imaging.

General off-axis imaging can be considered in terms of the field position-dependent aberrations of coma, astigma-

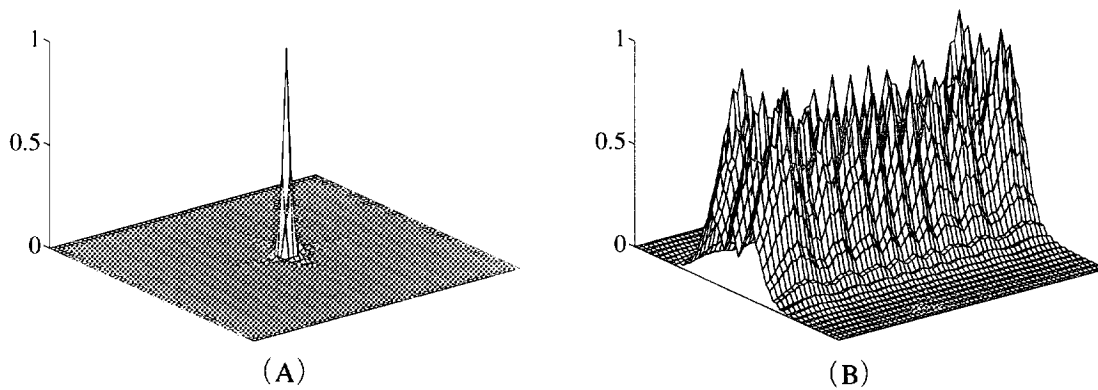


Fig. 4. Off-axis PSFs of a coma-corrected lens system. (A) represents the on-axis PSF. (B) represents off-axis PSF. Large uncorrected off-axis aberrations result in a greatly increased PSF size.

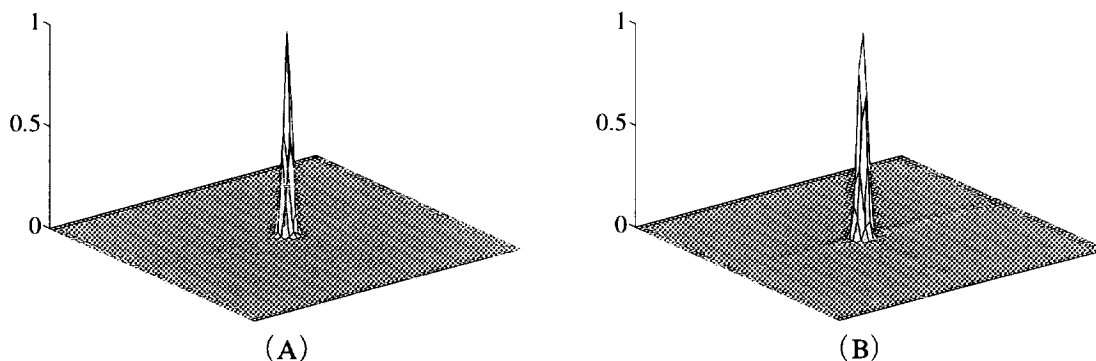


Fig. 5. Off-axis system PSFs of an optical/digital aberration-tolerant coma-corrected lens system. (A) represents on-axis PSF. (B) represents off-axis PSF. Uncorrected off-axis aberrations do not increase the size of the system PSF since the system is invariant to these aberrations.

tism, field curvature, and distortion.⁷⁾ We have specifically assumed the use of a coma-corrected lens systems. It is well known that even single lens elements can be designed to be coma-free. Distortion is merely a deterministic shifting of the image point and is easily corrected with post-processing, and so will be ignored. Therefore, the main contributing off-axis aberrations are astigmatism and field curvature. Figure 4 is the result of simulating PSFs of a two-dimensional coma-corrected system on-axis (A), and off-axis (B). The size of the off-axis PSF, even in a coma-corrected system, can many times larger than the size of the on-axis PSF. Further correction of the remaining off-axis aberrations can result in more optical elements and a more costly optical design.

Figure 5 represents aberration-tolerant off-axis imaging with a coma-corrected system. The physical system is the same as the one used in Fig. 4 with the addition of a two-dimensional CPM optical mask. This mask is mathematically described, in normalized coordinates, as $P(x,y) = \exp\{j\alpha(x^3+y^3)\}$, $|x| \leq 1$, $|y| \leq 1$, and with $\alpha = 90$ and $j = \sqrt{-1}$. The resulting optical/digital system PSF, after digital filtering, when on-axis is given in Fig. 5(A). The resulting off-axis system PSF is given in Fig. 5(B). The off-axis parameters of the physical system are identical to those of Fig. 4(B). The aberration-tolerant system PSF shows very little change as a function of field position. The off-axis system PSF is nearly identical to that on-axis, and

also nearly identical to that of the on-axis PSF from the traditional system. Thus, the focus-invariant system also acts as a aberration-tolerant system.

4. Conclusions

We have shown through simulation, and through experiments, that important lens aberrations can be controlled with focus-invariant optical/digital systems. Such systems are then also aberration-tolerant optical/digital systems. Aberration-tolerant systems are substantially insensitive to many unknown or unmeasured lens aberrations.

Acknowledgments

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References

- 1) E.R. Dowski, Jr. and W.T. Cathey: *Appl. Opt.* 34 (1995) 1859.
- 2) E.R. Dowski, W.T. Cathey and A.R. FitzGerrell: *Proc. SPIE* 2730 (1995) 120.
- 3) J. van der Gracht, E.R. Dowski, W.T. Cathey and J.P. Bowen: *Proc. SPIE* 2537 (1995) 279.
- 4) K. Brenner, A. Lohmann and J. Ojeda-Castañeda: *Opt. Commun.* 44 (1983) 323.
- 5) C.E. Cook and M. Bernfeld: *Radar Signals* (Academic Press, New York, 1967) Chap. 4, p. 59.
- 6) A.W. Rihaczek: *Principles of High Resolution Radar* (McGraw-Hill, New York, 1969) Chap. 4 and 5, p. 87.
- 7) J.E. Harvey: *Am. J. Phys.* 47 (1979) 974.